

4.5 Fault Analysis:

Under normal conditions, a power system operates under balanced conditions with all equipments carrying normal load currents and the bus voltages within the prescribed limits. This condition can be disrupted due to a fault in the system. A fault in a circuit is a failure that interferes with the normal flow of current. A short circuit fault occurs when the insulation of the system fails resulting in low impedance path either between phases or phase(s) to ground. This causes excessively high currents to flow in the circuit, requiring the operation of protective equipments to prevent damage to equipment. The short circuit faults can be classified as:

- Symmetrical faults
- Unsymmetrical faults

4.6 Symmetrical faults:

A three phase symmetrical fault is caused by application of three equal fault impedances \bar{Z}_f to the three phases, as shown in Fig. 4.39. If $\bar{Z}_f = 0$ the fault is called a solid or a bolted fault. These faults can be of two types: (a) line to line to line to ground fault (**LLLG** fault) or (b) line to line to line fault (**LLL** fault). Since the three phases are equally affected, the system remains balanced. That is why, this fault is called a symmetrical or a balanced fault and the fault analysis is done on per phase basis. The behaviour of **LLLG** fault and **LLL** fault is identical due to the balanced nature of the fault. This is a very severe fault that can occur in a system and if $\bar{Z}_f = 0$, this is usually the most severe fault that can occur in a system. Fortunately, such faults occur infrequently and only about 5% of the system faults are three phase faults.

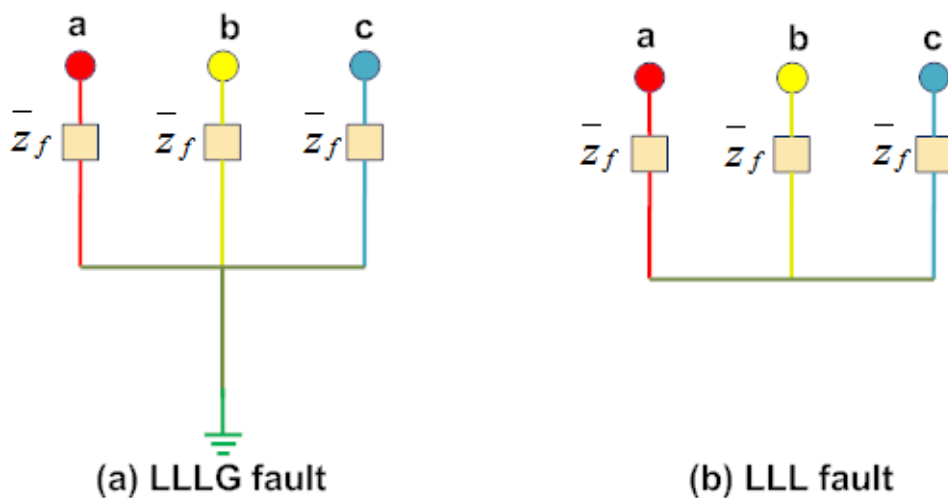


Figure 4.39: Symmetrical Fault

4.7 Unsymmetrical faults:

Faults in which the balanced state of the network is disturbed are called unsymmetrical or unbalanced faults. The most common type of unbalanced fault in a system is a single line to ground fault (**LG fault**). Almost 60 to 75% of faults in a system are **LG faults**. The other types of unbalanced faults are line to line faults (**LL faults**) and double line to ground faults (**LLG faults**). About 15 to 25% faults are **LLG faults** and 5 to 15% are **LL faults**. These faults are shown in Fig. 4.40.

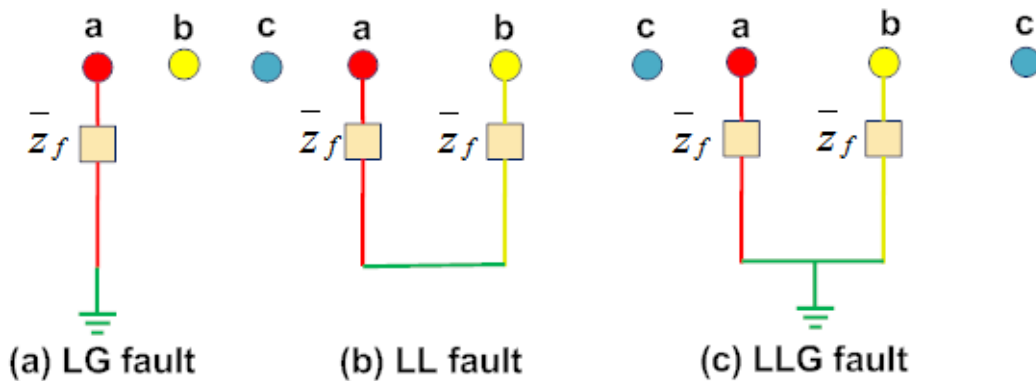


Figure 4.40: **Unsymmetrical Fault**

Majority of the faults occur on transmission lines as they are exposed to external elements. Lightning strokes may cause line insulators to flashover, high velocity winds may cause tower failure, ice loading and wind may result in mechanical failure of line or insulator and tree branches may cause short circuit. Much less common are the faults on cables, circuit breakers, generators, motors and transformers.

Fault analysis is necessary for selecting proper circuit breaker rating and for relay settings and coordination. The symmetrical faults are analysed on per phase basis while the unsymmetrical faults are analyzed using symmetrical components. Further, the \bar{Z}_{BUS} matrix is very useful for short circuit studies .

4.8 Symmetrical or Balanced three phase fault analysis:

In this type of faults all three phases are simultaneously short circuited. Since the network remains balanced, it is analyzed on per phase basis. The other two phases carry identical currents but with a phase shift of 120° . A fault in the network is simulated by connecting impedances in the network at the fault location. The faulted network is then solved using Thevenin's equivalent network as seen from the fault point. The bus impedance matrix is convenient to use for fault studies as its diagonal elements are the Thevenin's impedance of the network as seen from different buses. Prior to the occurrence of fault, the system is assumed to be in a balanced steady state and hence per phase network model is used. The generators are represented by a constant voltage source behind a suitable reactance which may be sub-transient, transient or normal d-axis reactance. The transmission lines are represented by their π -models with all impedances referred to a common base. A typical bus

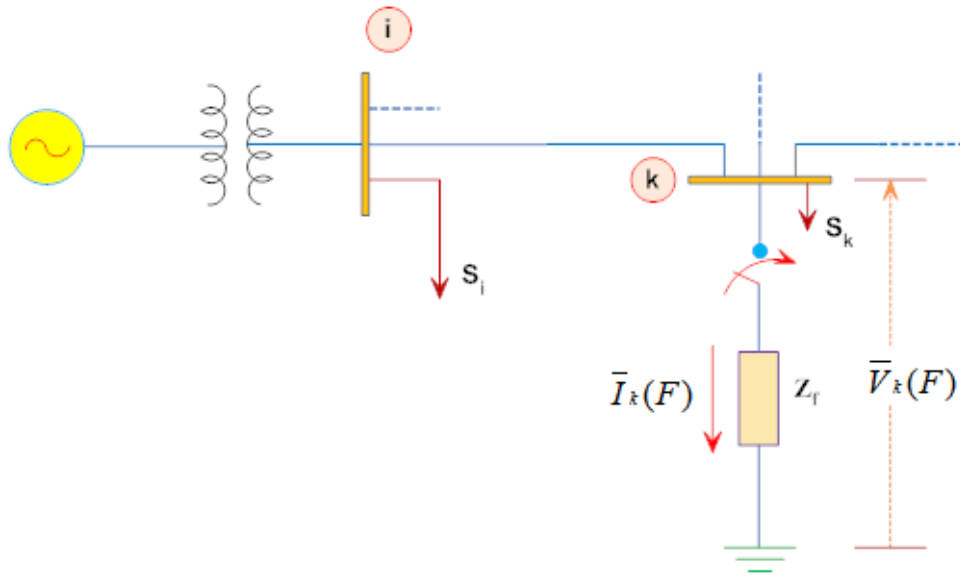


Figure 4.41: Fault at k^{th} bus of a power system

of an n - bus power system network is shown in Fig. 4.41. Further, a balanced three phase fault, through a fault impedance \bar{Z}_f is assumed to occur at k^{th} bus as shown in the figure. A pre-fault load flow provides the information about the pre-fault bus voltage.

Let $[\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{0})]$ be the prefault bus voltage vector $= [\bar{V}_1(0) \dots \bar{V}_k(0) \dots \bar{V}_n(0)]^T$ p.u. The fault at k^{th} bus through an impedance \bar{Z}_f will cause a change in the voltage of all the buses $[\Delta \bar{\mathbf{V}}_{\text{BUS}}]$ due to the flow of heavy currents through the transmission lines. This change can be calculated by applying a voltage $\bar{V}_k(0)$ at k^{th} bus and short circuiting all other voltage sources. The sources and loads are replaced by their equivalent impedances. This is shown in Fig. 4.42. In Fig. 4.42, \bar{Z}_i

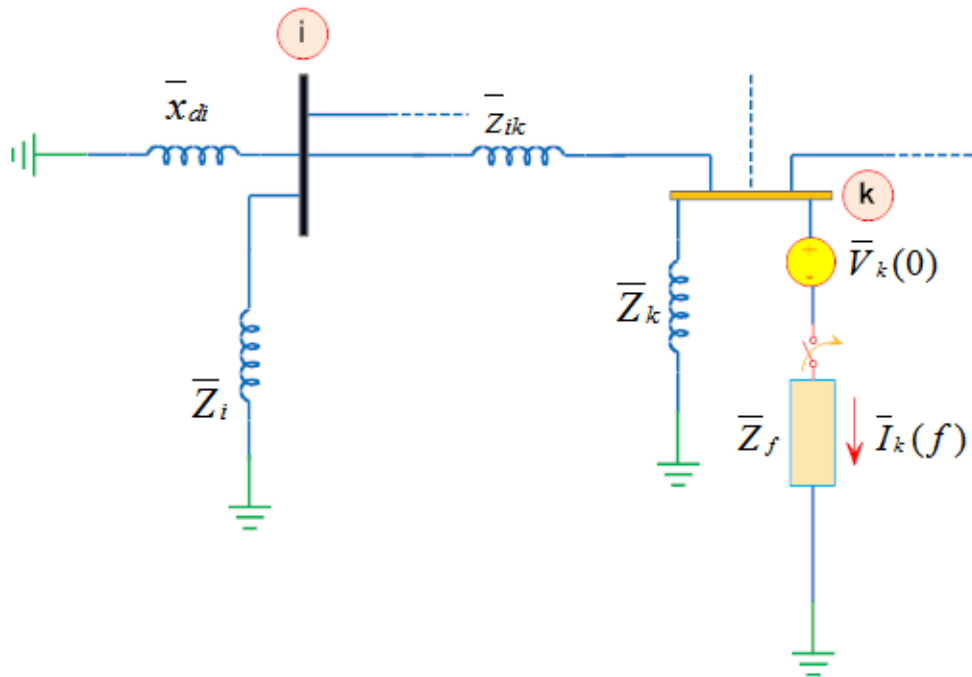


Figure 4.42: Network representation for calculating $[\Delta \bar{\mathbf{V}}_{\text{BUS}}]$

and \bar{Z}_k are the equivalent load impedances as bus i and k respectively, \bar{z}_{ik} is the impedance of line

between i^{th} and k^{th} buses. \bar{x}_{di} is the appropriate generator reactance, \bar{Z}_f is the fault impedance, $\bar{I}_k(F)$ is the fault current and $\bar{V}_k(0)$ is the prefault voltage at k^{th} bus. From the superposition theorem, the bus voltages due to a fault can be obtained as the sum of prefault bus voltages and the change in bus voltages due to fault, i.e.,

$$[\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{F})] = [\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{0})] + [\Delta \bar{\mathbf{V}}_{\text{BUS}}] \quad (4.64)$$

where,

$$[\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{F})] = \text{Vector of bus voltages during fault} = [\bar{V}_1(F) \dots \bar{V}_i(F) \dots \bar{V}_n(F)]^T$$

$$[\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{0})] = \text{Vector of pre-fault bus voltages} = [\bar{V}_1(0) \dots \bar{V}_i(0) \dots \bar{V}_n(0)]^T$$

$$[\Delta \bar{\mathbf{V}}_{\text{BUS}}] = \text{Vector of change in bus voltages due to fault} = [\Delta \bar{V}_1 \dots \Delta \bar{V}_k \dots \Delta \bar{V}_n]^T$$

Also the bus injected current $[\bar{\mathbf{I}}_{\text{BUS}}]$ can be expressed as,

$$[\bar{\mathbf{I}}_{\text{BUS}}] = [\bar{\mathbf{Y}}_{\text{BUS}}] [\bar{\mathbf{V}}_{\text{BUS}}] \quad (4.65)$$

where, $[\bar{\mathbf{V}}_{\text{BUS}}]$ is the bus voltage vector and $[\bar{\mathbf{Y}}_{\text{BUS}}]$ is the bus admittance matrix.

With all the bus currents, except of the faulted bus k , equal to zero, the node equation for the network of Fig. 4.42 can be written as

$$\begin{bmatrix} 0 \\ \vdots \\ -\bar{I}_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \dots & \bar{Y}_{1k} & \dots & \bar{Y}_{1n} \\ \vdots & & \vdots & & \vdots \\ \bar{Y}_{k1} & \dots & \bar{Y}_{kk} & \dots & \bar{Y}_{kn} \\ \vdots & & \vdots & & \vdots \\ \bar{Y}_{n1} & \dots & \bar{Y}_{nk} & \dots & \bar{Y}_{nn} \end{bmatrix} \begin{bmatrix} \Delta \bar{V}_1 \\ \vdots \\ \Delta \bar{V}_k \\ \vdots \\ \Delta \bar{V}_n \end{bmatrix} \quad (4.66)$$

As the fault current $\bar{I}_k(F)$ is leaving the bus it is taken as a negative current entering the bus. Hence,

$$[\bar{\mathbf{I}}_{\text{BUS}}(\mathbf{F})] = [\bar{\mathbf{Y}}_{\text{BUS}}] [\Delta \bar{\mathbf{V}}_{\text{BUS}}] \quad (4.67)$$

$[\Delta \bar{\mathbf{V}}_{\text{BUS}}]$ can be calculated as:

$$[\Delta \bar{\mathbf{V}}_{\text{BUS}}] = [\bar{\mathbf{Y}}_{\text{BUS}}]^{-1} [\bar{\mathbf{I}}_{\text{BUS}}(\mathbf{F})] = [\bar{\mathbf{Z}}_{\text{BUS}}] [\bar{\mathbf{I}}_{\text{BUS}}(\mathbf{F})] \quad (4.68)$$

where, $[\bar{\mathbf{Z}}_{\text{BUS}}]$ is the bus impedance matrix = $[\bar{\mathbf{Y}}_{\text{BUS}}]^{-1}$.

Substituting the expression of $[\Delta \bar{\mathbf{V}}_{\text{BUS}}]$ from equation (4.68) in equation (4.64) one can write,

$$[\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{F})] = [\bar{\mathbf{V}}_{\text{BUS}}(\mathbf{0})] + [\bar{\mathbf{Z}}_{\text{BUS}}(\mathbf{F})] [\bar{\mathbf{I}}_{\text{BUS}}(\mathbf{F})] \quad (4.69)$$

Expanding the above equation one can write,

$$\begin{bmatrix} \bar{V}_1(F) \\ \vdots \\ \bar{V}_k(F) \\ \vdots \\ \bar{V}_n(F) \end{bmatrix} = \begin{bmatrix} \bar{V}_1(0) \\ \vdots \\ \bar{V}_k(0) \\ \vdots \\ \bar{V}_n(0) \end{bmatrix} + \begin{bmatrix} \bar{Z}_{11} & \cdots & \bar{Z}_{1k} & \cdots & \bar{Z}_{1n} \\ \vdots & & \vdots & & \vdots \\ \bar{Z}_{k1} & \cdots & \bar{Z}_{kk} & \cdots & \bar{Z}_{kn} \\ \vdots & & \vdots & & \vdots \\ \bar{Z}_{n1} & \cdots & \bar{Z}_{nk} & \cdots & \bar{Z}_{nn} \end{bmatrix} \begin{bmatrix} (0) \\ \vdots \\ -\bar{I}_k(F) \\ \vdots \\ 0 \end{bmatrix} \quad (4.70)$$

The bus voltage of k^{th} bus can be expressed as:

$$\bar{V}_k(F) = \bar{V}_k(0) - \bar{Z}_{kk}\bar{I}_k(F) \quad (4.71)$$

Also from Fig. 4.41

$$\bar{V}_k(F) = \bar{Z}_F\bar{I}_k(F) \quad (4.72)$$

For a bolted fault $\bar{Z}_f = 0$ and hence, $\bar{V}_k(F) = 0$. Thus the fault current $\bar{I}_k(F)$ for bolted fault can be expressed using equation (4.71) as,

$$\boxed{\bar{I}_k(F) = \frac{\bar{V}_k(0)}{\bar{Z}_{kk}}} \quad (4.73)$$

For faulty with non-zero fault impedance \bar{Z}_f , the fault current can be calculated as:

$$\boxed{\bar{I}_k(F) = \frac{\bar{V}_k(0)}{\bar{Z}_{kk} + \bar{Z}_f}} \quad (4.74)$$

The quantity \bar{Z}_{kk} in equation (4.73) and equation (4.74) is the Thevenin's impedance or open-circuit impedance of the network as seen from the faulted bus k . From equation (4.70), the bus voltage after fault for the unfaulted or healthy buses can be written as:

$$\bar{V}_i(F) = \bar{V}_i(0) - \bar{Z}_{ik}\bar{I}_k(F) \quad \forall i = 1, 2, \dots, n, \quad i \neq k \quad (4.75)$$

Substituting $\bar{I}_k(F)$ from equation (4.73), $\bar{V}_i(F)$ can be expressed as:

$$\boxed{\bar{V}_i(F) = \bar{V}_i(0) - \frac{\bar{Z}_{ik}}{\bar{Z}_{kk} + \bar{Z}_f}\bar{V}_k(0)} \quad (4.76)$$

The fault current $\bar{I}_{ij}(F)$ flowing in the line connecting i^{th} and j^{th} bus can be calculated as

$$\boxed{\bar{I}_{ij}(F) = \frac{\bar{V}_i(F) - \bar{V}_j(F)}{\bar{z}_{ij}}} \quad (4.77)$$

where \bar{z}_{ij} is the impedance of line connecting buses i and j .

4.9 Unsymmetrical or Unbalanced fault analysis:

For the analysis of unsymmetrical or unbalanced faults, symmetrical component method is used. The use of symmetrical components simplifies the analysis procedure of unbalanced system and also helps in improving the understanding of the system behavior during fault conditions.

A review of symmetrical components is presented next.

4.9.1 Symmetrical components:

Any unbalanced set of three phase voltage or current phasors can be replaced by three balanced sets of three phase voltage or current phasors. These three balanced set of voltage or current phasors are called symmetrical components of voltages or currents. Let \bar{I}_a , \bar{I}_b , and \bar{I}_c be an arbitrary set of three current phasors representing phase currents. Then using symmetrical components they can be expressed as:

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} \bar{I}_{a0} \\ \bar{I}_{b0} \\ \bar{I}_{c0} \end{bmatrix} + \begin{bmatrix} \bar{I}_{a1} \\ \bar{I}_{b1} \\ \bar{I}_{c1} \end{bmatrix} + \begin{bmatrix} \bar{I}_{a2} \\ \bar{I}_{b2} \\ \bar{I}_{c2} \end{bmatrix} \quad (4.78)$$

Or,

$$[\bar{\mathbf{I}}]_{\text{abc}} = [\bar{\mathbf{I}}_0] + [\bar{\mathbf{I}}_1] + [\bar{\mathbf{I}}_2]$$

where,

$[\bar{\mathbf{I}}]_{\text{abc}} = [\bar{I}_a \ \bar{I}_b \ \bar{I}_c]^T$ is the arbitrary set of three current phasors of phase currents.

$[\bar{\mathbf{I}}_0] = [\bar{I}_{a0} \ \bar{I}_{b0} \ \bar{I}_{c0}]^T$ is the set of zero sequence components. The magnitudes of the three zero sequence components are equal i.e. $|\bar{I}_{a0}| = |\bar{I}_{b0}| = |\bar{I}_{c0}|$ and they are co-phasors.

$[\bar{\mathbf{I}}_1] = [\bar{I}_{a1} \ \bar{I}_{b1} \ \bar{I}_{c1}]^T$ is the set of positive sequence components, with $\bar{I}_{a1} = |\bar{I}_{a1}| \angle 0^\circ$, $\bar{I}_{b1} = |\bar{I}_{b1}| \angle -120^\circ$, and $\bar{I}_{c1} = |\bar{I}_{c1}| \angle 120^\circ$, with $|\bar{I}_{a1}| = |\bar{I}_{b1}| = |\bar{I}_{c1}|$.

$[\bar{\mathbf{I}}_2] = [\bar{I}_{a2} \ \bar{I}_{b2} \ \bar{I}_{c2}]^T$ is the set of negative sequence components, with $\bar{I}_{a2} = |\bar{I}_{a2}| \angle 0^\circ$, $\bar{I}_{b2} = |\bar{I}_{b2}| \angle 120^\circ$, and $\bar{I}_{c2} = |\bar{I}_{c2}| \angle -120^\circ$, with $|\bar{I}_{a2}| = |\bar{I}_{b2}| = |\bar{I}_{c2}|$

The graphical representation of the sequence components is shown in Fig. 4.43.

Let an operator ‘**a**’ be defined such that $\mathbf{a} = \angle 120^\circ$. Any phasor multiplied by ‘**a**’ undergoes a counter clockwise rotation of 120° without any change in the magnitude. Further,

$$\mathbf{a} = 1 \angle 120^\circ$$

$$\mathbf{a}^2 = 1 \angle 240^\circ$$

$$\mathbf{a}^3 = 1 \angle 360^\circ$$

$$\text{also } 1 + \mathbf{a} + \mathbf{a}^2 = 0$$

$$\bar{I}_{a1} = I_{a1} \angle \theta_1$$

where, $\angle \theta_1$ is the angle of phase ‘**a**’ positive sequence current.

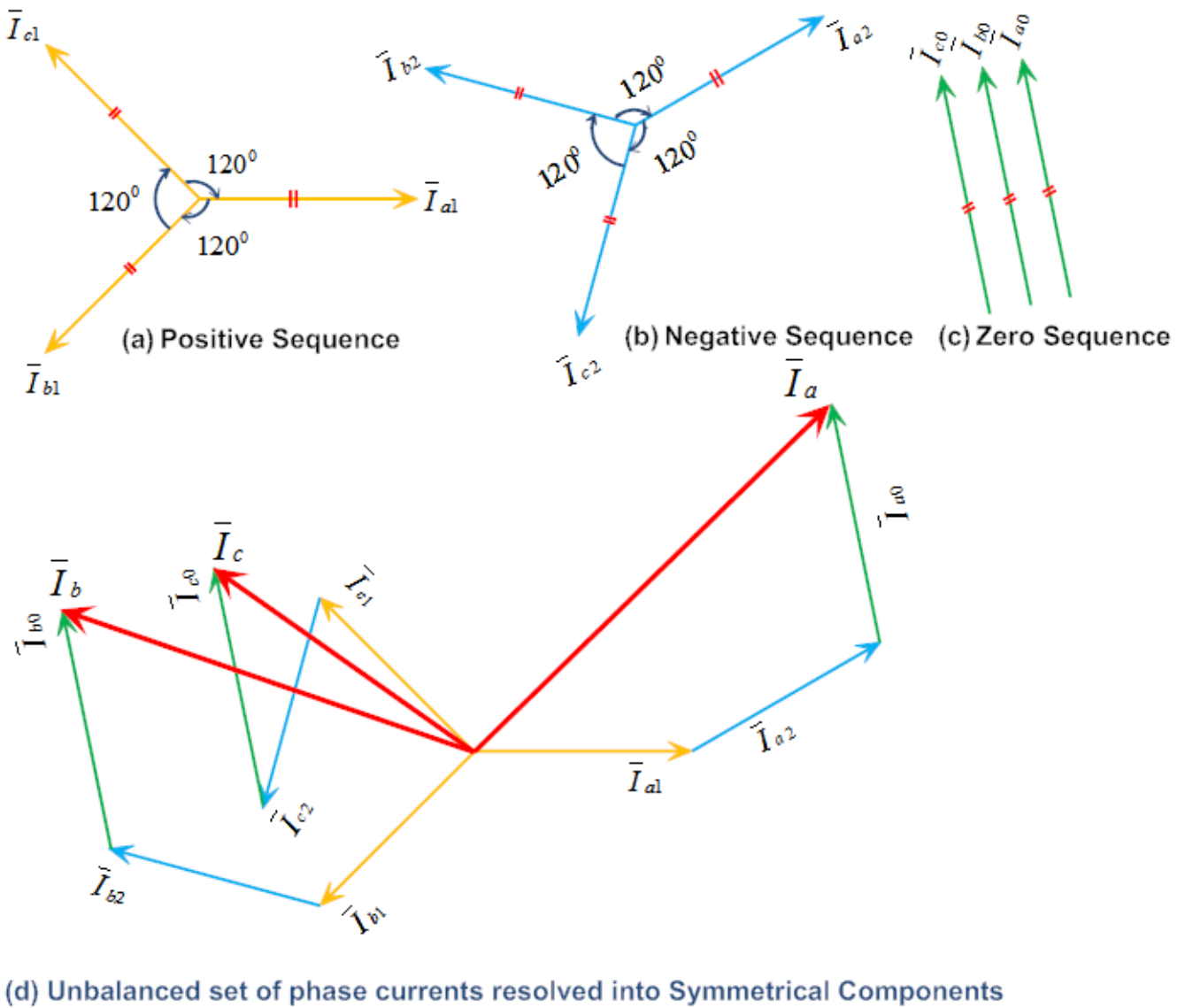


Figure 4.43: Representation of Symmetrical Components

$$\bar{I}_{b1} = \mathbf{a}^2 \bar{I}_{a1}$$

$$\bar{I}_{c1} = \mathbf{a} \bar{I}_{a1}$$

The phase sequence of the positive component set is 'abc'.

Similarly the negative sequence set can be written as:

$$\bar{I}_{a2} = I_{a2} \angle \theta_2$$

where, $\angle \theta_2$ is the angle of phase 'a' negative sequence current.

$$\bar{I}_{b2} = \mathbf{a} \bar{I}_{a2}$$

$$\bar{I}_{c2} = \mathbf{a}^2 \bar{I}_{a2}$$

The phase sequence of the negative component set is ‘acb’.

The zero-sequence component set can be written as:

$$\bar{I}_{a0} = \bar{I}_{a0} \angle \theta_0 = \bar{I}_{b0} = \bar{I}_{c0}$$

where, $\angle \theta_0$ is the angle of phase ‘a’ zero sequence current.

Hence, equation (4.78) can be simplified as:

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \bar{I}_{a0} \\ \bar{I}_{a1} \\ \bar{I}_{a2} \end{bmatrix} \quad (4.79)$$

It can also expressed in a compact form as:

$$[\bar{\mathbf{I}}]_{\text{abc}} = [\bar{\mathbf{A}}] [\bar{\mathbf{I}}]_{012} \quad (4.80)$$

where, $[\bar{\mathbf{I}}]_{\text{abc}}$ = set of phase quantities = $[\bar{I}_a \ \bar{I}_b \ \bar{I}_c]^T$

$[\bar{\mathbf{I}}]_{012}$ = set of sequence quantities = $[\bar{I}_{a0} \ \bar{I}_{a1} \ \bar{I}_{a2}]^T$

$\bar{\mathbf{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}$ is the symmetrical component transformation matrix.

The symmetrical components $[\bar{\mathbf{I}}]_{012}$ can be written in terms of phase quantities $[\bar{\mathbf{I}}]_{\text{abc}}$ as:

$$[\bar{\mathbf{I}}]_{012} = [\bar{\mathbf{A}}]^{-1} [\bar{\mathbf{I}}]_{\text{abc}} \quad (4.81)$$

where, $\bar{\mathbf{A}}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}$ thus,

$$\begin{aligned} \bar{I}_{a0} &= \frac{1}{3} [\bar{I}_a + \bar{I}_b + \bar{I}_c] \\ \bar{I}_{a1} &= \frac{1}{3} [\bar{I}_a + \mathbf{a}\bar{I}_b + \mathbf{a}^2\bar{I}_c] \\ \bar{I}_{a2} &= \frac{1}{3} [\bar{I}_a + \mathbf{a}^2\bar{I}_b + \mathbf{a}\bar{I}_c] \end{aligned} \quad (4.82)$$

To summarize:

- [For voltage:](#)

$$\boxed{[\bar{\mathbf{V}}]_{\text{abc}} = [\bar{\mathbf{A}}] [\bar{\mathbf{V}}]_{012}} \quad (4.83)$$

$$\boxed{[\bar{\mathbf{V}}]_{012} = [\bar{\mathbf{A}}]^{-1} [\bar{\mathbf{V}}]_{abc}} \quad (4.84)$$

where, $[\bar{\mathbf{V}}]_{abc}$ is the set of phase voltages, and $[\bar{\mathbf{V}}]_{012}$ is the set of sequence voltages.

- For current:

$$\boxed{[\bar{\mathbf{I}}]_{abc} = [\bar{\mathbf{A}}] [\bar{\mathbf{I}}]_{012}} \quad (4.85)$$

$$\boxed{[\bar{\mathbf{I}}]_{012} = [\bar{\mathbf{A}}]^{-1} [\bar{\mathbf{I}}]_{abc}} \quad (4.86)$$

where, $[\bar{\mathbf{I}}]_{abc}$ is the set of phase currents, and $[\bar{\mathbf{I}}]_{012}$ is the set of sequence currents.

Before starting unbalanced fault analysis, it is necessary to learn about the sequence networks of different power system components, which we will discuss in the next lecture.